

On the Achievability of Interference Alignment for Three-Cell Constant Cellular Interfering Networks

Yanjun Ma, Jiandong Li, *Senior Member, IEEE*, Rui Chen, *Student Member, IEEE*, and Qin Liu

Abstract—For a three-cell constant cellular interfering network, a new property of alignment is identified, i.e., interference alignment (IA) solution obtained in an user-cooperation scenario can also be applied in a non-cooperation environment. By using this property, an algorithm is proposed by jointly designing transmit and receive beamforming matrices. Analysis and numerical results show that more degree of freedom (DoF) can be achieved compared with conventional schemes in most cases.

Index Terms—Interference channel, interference alignment, degrees of freedom, Multi-User MIMO.

I. INTRODUCTION

INTERFERENCE alignment (IA) is a promising technique to mitigate interference in wireless communication systems. It was shown that $\frac{K}{2}$ DoF is achievable per time, frequency or antenna dimension in a K -user interference channel (IC) [1]. For a K -user constant MIMO IC, IA based schemes were introduced in [2]–[5], where it was shown that more DoF is achievable than that of conventional schemes. For a constant cellular interfering network, it was shown in [6] that their scheme provides respectable gain for a 19 hexagonal wrap-around-cell layout. However, interference-free DoF is only achievable for a two-cell layout. It was shown in [7] that optimal DoF is achievable when $\lceil \frac{3}{2}N \rceil \leq M < 2N$ for a two-cell MIMO interfering broadcast channel, where each transmitter is equipped with M antennas and each receiver is equipped with N antennas.

In this letter, we focus on a three-cell constant cellular interfering network by using a new property of alignment, i.e., *IA solution obtained in an user-cooperation scenario can also be applied in a non-cooperation environment*. We assume that each base station (BS) is equipped with M antennas and each mobile station (MS) is equipped with N antennas, and $M > N$ which is most possible in a practical environment. We also assume there are K cell-edge users per cell where $K > 1$, and each user sends d streams to its served BS simultaneously. We show that totally $3Kd$ DoF is achievable if $M = KN$ and $d \leq \lfloor \frac{M}{3K-1} \rfloor$ or if $M < KN$ and $d \leq \min \{ \lfloor \frac{M}{3K-1} \rfloor, 3(KN - M) \}$. Numerical results show that more DoF can be achieved compared with conventional schemes in most cases.

This work was supported in part by the National Science Fund for Distinguished Young Scholars under Grant 60725105, by the National Basic Research Program of China under Grant 2009CB320404, by the Program for Changjiang Scholars and Innovative Research Team in University under Grant IRT0852, by the Key Project of Chinese Ministry of Education under Grant 107103, and by the 111 Project under Grant B08038.

The authors are with the State Key Laboratory of Integrated Service Networks, Xidian University, Xi'an 710071, China (e-mail: {ymj, jdli, qinliu}@mail.xidian.edu.cn, rchenxidian@gmail.com).

II. SYSTEM MODEL

For a three-cell constant cellular interfering network (an example is shown in Fig. 1), we assume that each BS is equipped with M antennas, each MS is equipped with N antennas, and there are K cell-edge users per cell. For notation convenience, we refer to the j -th user in the i -th cell as user $[i, j]$. For ease of analysis, we consider an uplink scenario¹, and assume that each user tries to convey d data streams to its served BS by using a normalized precoding matrix $\mathbf{W}^{[ij]}$. Then we have

$$\mathbf{x}^{[ij]} = \mathbf{W}^{[ij]} \mathbf{s}^{[ij]}, \quad (1)$$

where $\mathbf{s}^{[ij]}$ is a $d \times 1$ vector, which denotes the transmitted data streams from user $[i, j]$, and satisfies an average power constraint, i.e., $\mathbb{E} [\|\mathbf{s}^{[ij]}\|^2] \leq P$. The received signal at the i -th BS is represented as

$$\mathbf{y}^{[i]} = \sum_{k=1}^3 \sum_{j=1}^K \mathbf{H}_i^{[kj]} \mathbf{W}^{[kj]} \mathbf{s}^{[kj]} + \mathbf{n}^{[i]}, \quad (2)$$

where $\mathbf{n}^{[i]} \sim \mathcal{N}(\mathbf{0}, \sigma^2 \mathbf{I})$ is the $M \times 1$ additive white Gaussian noise, and $\mathbf{H}_i^{[kj]}$ is the $M \times N$ channel matrix from user $[k, j]$ to the i -th BS. The channel is assumed to be constant over time, and perfect channel state information (CSI) is available at all BSs and MSs. The i -th BS decodes the desired signal for user $[i, j]$ by multiplying the cascaded receive beamforming matrices, $\mathbf{V}^{[i]}$ and $\mathbf{P}_j^{[i]}$, and we obtain the desired signal for user $[i, j]$

$$\tilde{\mathbf{y}}^{[ij]} = \mathbf{P}_j^{[i]\dagger} \mathbf{V}^{[i]\dagger} \sum_{k=1}^3 \sum_{j=1}^K \mathbf{H}_i^{[kj]} \mathbf{W}^{[kj]} \mathbf{s}^{[kj]} + \tilde{\mathbf{n}}^{[ij]}, \quad (3)$$

where $\mathbf{V}^{[i]}$ is the normalized inter-cell interference (ICI) elimination matrix, $\mathbf{P}_j^{[i]}$ is the normalized inter-user interference (IUI) elimination matrix, and $\tilde{\mathbf{n}}^{[ij]} = \mathbf{P}_j^{[i]\dagger} \mathbf{V}^{[i]\dagger} \mathbf{n}^{[i]}$ is the effective noise vector. The notation $(\cdot)^\dagger$ stands for conjugate transpose. Let $\mathbf{P}^{[i]} = \{\mathbf{P}_1^{[i]}, \dots, \mathbf{P}_K^{[i]}\}$ denote the combined IUI elimination matrix at the i -th BS. We define the DoF region as the following [1]:

$$\mathcal{D} = \left\{ (d^{[11]}, \dots, d^{[3K]}) \in \mathbb{R}_+^{3K} \mid \forall (\omega_{11}, \dots, \omega_{3K}) \in \mathbb{R}_+^{3K}, \right. \\ \left. \sum_{i=1}^3 \sum_{j=1}^K \omega_{ij} d^{[ij]} \leq \limsup_{\text{SNR} \rightarrow \infty} \left[\sup_{\mathbf{R} \in \mathcal{C}} \frac{1}{\log \text{SNR}} \sum_{i=1}^3 \sum_{j=1}^K \omega_{ij} R^{[ij]} \right] \right\}, \quad (4)$$

¹By using the reciprocity of alignment [5], our scheme can also be applied in a downlink scenario.

where \mathcal{D} is the capacity region, $\text{SNR} = P/\sigma^2$, and $R^{[ij]}$ is the rate of user $[i, j]$. Let

$$\eta = \sum_{i \in \{1,2,3\}} \sum_{j \in \{1,\dots,K\}} d^{[ij]} \quad (5)$$

be the total DoF in the network.

III. AN IA BASED SCHEME FOR THREE-CELL CONSTANT CELLULAR INTERFERING NETWORKS

In this section, an IA based scheme is introduced for the three-cell constant cellular interfering network. A motivating example is given first (as is shown in Fig. 1), where $M = 16$, $N = 8$, $K = 2$, and $d = 3$. We show that totally 18 DoF is achievable in this scenario.

We divide our scheme into two phases. First, ICI is aligned into a smaller vector space at each BS by joint design of all the precoding matrices. Second, IUI is eliminated through cascaded receive beamforming matrices at each BS.

Phase I: ICI alignment. By applying the IA solution obtained in an MIMO IC to a cellular environment, which is presented in Fig. 1, we show that 12 ICI streams at each BS can be aligned into a vector space of 9 dimensions simultaneously, i.e.,

$$\dim \left\{ \text{span} \left(\begin{bmatrix} \mathbf{H}_1^{[21]} \mathbf{W}^{[21]} & \mathbf{H}_1^{[22]} \mathbf{W}^{[22]} \\ \mathbf{H}_1^{[31]} \mathbf{W}^{[31]} & \mathbf{H}_1^{[32]} \mathbf{W}^{[32]} \end{bmatrix} \right) \right\} = 9, \quad (6)$$

$$\dim \left\{ \text{span} \left(\begin{bmatrix} \mathbf{H}_2^{[11]} \mathbf{W}^{[11]} & \mathbf{H}_2^{[12]} \mathbf{W}^{[12]} \\ \mathbf{H}_2^{[31]} \mathbf{W}^{[31]} & \mathbf{H}_2^{[32]} \mathbf{W}^{[32]} \end{bmatrix} \right) \right\} = 9, \quad (7)$$

$$\dim \left\{ \text{span} \left(\begin{bmatrix} \mathbf{H}_3^{[11]} \mathbf{W}^{[11]} & \mathbf{H}_3^{[12]} \mathbf{W}^{[12]} \\ \mathbf{H}_3^{[21]} \mathbf{W}^{[21]} & \mathbf{H}_3^{[22]} \mathbf{W}^{[22]} \end{bmatrix} \right) \right\} = 9. \quad (8)$$

Let $\mathbf{W}^{[i]} = [\mathbf{W}^{[i1]\dagger} \ \mathbf{W}^{[i2]\dagger}]^\dagger$ be the combined transmit precoding matrix of all cell-edge users in the i -th cell. Let $\mathbf{G}^{[ij]} = [\mathbf{H}_i^{[j1]} \ \mathbf{H}_i^{[j2]}]$ represent the combined channel matrix. Then the effective channel is a three-user MIMO IC where each node is equipped with $M = 16$ antennas. Following the analysis in [1], there exists a 16×8 $\mathbf{W}^{[i]}$, $i = \{1, 2, 3\}$, satisfies (9) - (11).

$$\text{span} [\mathbf{G}^{[12]} \mathbf{W}^{[2]}] = \text{span} [\mathbf{G}^{[13]} \mathbf{W}^{[3]}], \quad (9)$$

$$\text{span} [\mathbf{G}^{[21]} \mathbf{W}^{[1]}] = \text{span} [\mathbf{G}^{[23]} \mathbf{W}^{[3]}], \quad (10)$$

$$\text{span} [\mathbf{G}^{[31]} \mathbf{W}^{[1]}] = \text{span} [\mathbf{G}^{[32]} \mathbf{W}^{[2]}]. \quad (11)$$

Let

$$\mathbf{E} = (\mathbf{G}^{[31]})^{-1} \mathbf{G}^{[32]} (\mathbf{G}^{[12]})^{-1} \mathbf{G}^{[13]} (\mathbf{G}^{[23]})^{-1} \mathbf{G}^{[21]}, \quad (12)$$

$$\mathbf{F} = (\mathbf{G}^{[32]})^{-1} \mathbf{G}^{[31]}, \quad (13)$$

$$\mathbf{C} = (\mathbf{G}^{[23]})^{-1} \mathbf{G}^{[21]}. \quad (14)$$

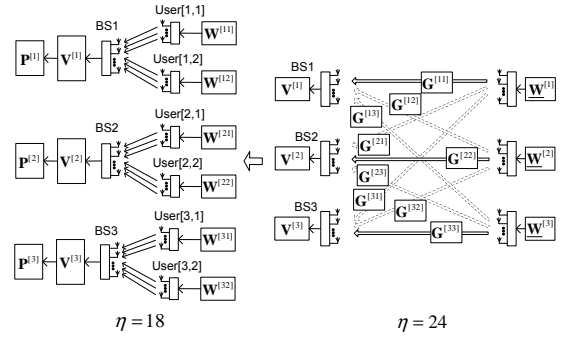


Fig. 1. An illustration for a three-cell constant cellular interfering network, where $M = 16$, $N = 8$, $K = 2$, and $d = 3$.

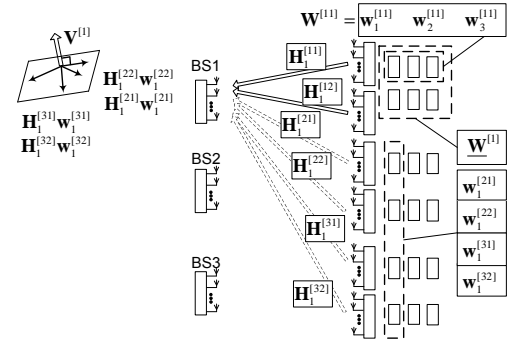


Fig. 2. In this illustration, it is shown that 4 interference signals are aligned into a vector space of 3 dimensions.

Then

$$\text{span}(\mathbf{W}^{[1]}) = \text{span}(\mathbf{E} \mathbf{W}^{[1]}), \quad (15)$$

$$\mathbf{W}^{[2]} = \mathbf{F} \mathbf{W}^{[1]}, \quad (16)$$

$$\mathbf{W}^{[3]} = \mathbf{C} \mathbf{W}^{[1]}. \quad (17)$$

So, if we allow full user-cooperation, 24 DoF is achievable in this scenario. However, if user-cooperation is not allowed, we show that (6) - (8) are also satisfied in the following.

Let $\{\mathbf{e}_1, \dots, \mathbf{e}_{16}\}$ be the eigenvectors of \mathbf{E} . Let $\mathbf{W}^{[1]} = \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ be any three eigenvectors of \mathbf{E} , for example, let $\mathbf{W}^{[1]} = \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$, and calculate $\mathbf{W}^{[2]}$ and $\mathbf{W}^{[3]}$ according to (16) and (17), respectively. Let $\mathbf{W}^{[ij]} = [\mathbf{w}_1^{[ij]} \ \mathbf{w}_2^{[ij]} \ \mathbf{w}_3^{[ij]}]$, $i \in \{1, 2, 3\}$, $j \in \{1, 2\}$ (Fig. 2 shows the relationship among $\mathbf{W}^{[i]}$, $\mathbf{W}^{[ij]}$, and $\mathbf{w}_k^{[ij]}$). We rewrite (9) as

$$\text{span} \left(\begin{bmatrix} \mathbf{H}_1^{[21]} & \mathbf{H}_1^{[22]} \\ \mathbf{H}_1^{[31]} & \mathbf{H}_1^{[32]} \end{bmatrix} \begin{bmatrix} \mathbf{w}_k^{[21]} \\ \mathbf{w}_k^{[22]} \end{bmatrix} \right) = \text{span} \left(\begin{bmatrix} \mathbf{H}_1^{[31]} & \mathbf{H}_1^{[32]} \end{bmatrix} \begin{bmatrix} \mathbf{w}_k^{[31]} \\ \mathbf{w}_k^{[32]} \end{bmatrix} \right),$$

where $k \in \{1, 2, 3\}$, i.e.,

$$\text{span}(\mathbf{H}_1^{[21]} \mathbf{w}_k^{[21]} + \mathbf{H}_1^{[22]} \mathbf{w}_k^{[22]}) = \text{span}(\mathbf{H}_1^{[31]} \mathbf{w}_k^{[31]} + \mathbf{H}_1^{[32]} \mathbf{w}_k^{[32]}).$$

So, there exist non-zero α_{1k} and α_{2k} satisfy

$$\alpha_{1k}(\mathbf{H}_1^{[21]} \mathbf{w}_k^{[21]} + \mathbf{H}_1^{[22]} \mathbf{w}_k^{[22]}) = \alpha_{2k}(\mathbf{H}_1^{[31]} \mathbf{w}_k^{[31]} + \mathbf{H}_1^{[32]} \mathbf{w}_k^{[32]}),$$

i.e.,

$$\dim\left\{\text{span}\left(\begin{bmatrix} \mathbf{H}_1^{[21]} \mathbf{w}_k^{[21]} & \mathbf{H}_1^{[22]} \mathbf{w}_k^{[22]} \\ \mathbf{H}_1^{[31]} \mathbf{w}_k^{[31]} & \mathbf{H}_1^{[32]} \mathbf{w}_k^{[32]} \end{bmatrix}\right)\right\} = 3. \quad (18)$$

When the channels are generic, i.e., the elements of the channel matrices are randomly and independently generated from continuous distributions, we have

$$\begin{aligned} & \dim\left\{\text{span}\left(\begin{bmatrix} \mathbf{H}_1^{[21]} \mathbf{W}^{[21]} & \mathbf{H}_1^{[22]} \mathbf{W}^{[22]} & \mathbf{H}_1^{[31]} \mathbf{W}^{[31]} & \mathbf{H}_1^{[32]} \mathbf{W}^{[32]} \end{bmatrix}\right)\right\} \\ &= \dim\left\{\text{span}\left(\begin{bmatrix} \mathbf{H}_1^{[21]} \mathbf{w}_1^{[21]} & \mathbf{H}_1^{[22]} \mathbf{w}_1^{[22]} & \mathbf{H}_1^{[31]} \mathbf{w}_1^{[31]} & \mathbf{H}_1^{[32]} \mathbf{w}_1^{[32]} \end{bmatrix}\right)\right\} \\ &+ \dim\left\{\text{span}\left(\begin{bmatrix} \mathbf{H}_1^{[21]} \mathbf{w}_2^{[21]} & \mathbf{H}_1^{[22]} \mathbf{w}_2^{[22]} & \mathbf{H}_1^{[31]} \mathbf{w}_2^{[31]} & \mathbf{H}_1^{[32]} \mathbf{w}_2^{[32]} \end{bmatrix}\right)\right\} \\ &+ \dim\left\{\text{span}\left(\begin{bmatrix} \mathbf{H}_1^{[21]} \mathbf{w}_3^{[21]} & \mathbf{H}_1^{[22]} \mathbf{w}_3^{[22]} & \mathbf{H}_1^{[31]} \mathbf{w}_3^{[31]} & \mathbf{H}_1^{[32]} \mathbf{w}_3^{[32]} \end{bmatrix}\right)\right\} \\ &= 9. \end{aligned} \quad (19)$$

All the interference at BS1 has been aligned into a vector space of 9 dimensions. Along the same way, (7) and (8) are also satisfied.

Remark: When the channels are generic, \mathbf{e}_k will be a random vector, so are $\mathbf{w}_k^{[i1]}$ and $\mathbf{w}_k^{[i2]}$, where $\mathbf{e}_k = [\mathbf{w}_k^{[i1]\dagger} \ \mathbf{w}_k^{[i2]\dagger}]^\dagger$. Then $\mathbf{W}^{[ij]} = [\mathbf{w}_1^{[ij]} \ \mathbf{w}_2^{[ij]} \ \mathbf{w}_3^{[ij]}]$ will be full rank, i.e., $\text{rank}(\mathbf{W}^{[ij]}) = 3$, with probability 1. Fig. 3 shows rank distribution of $\mathbf{W}^{[ij]}$ at user $[i, j]$.

There are 7 interference-free dimensions left at each BS, and each BS can decode 6 streams sent from its cell-edge users. Then we choose

$$\begin{aligned} \mathbf{V}^{[1]} &\subseteq \text{null}\left(\begin{bmatrix} \mathbf{H}_1^{[21]} \mathbf{W}^{[21]} & \mathbf{H}_1^{[22]} \mathbf{W}^{[22]} & \mathbf{H}_1^{[31]} \mathbf{W}^{[31]} & \mathbf{H}_1^{[32]} \mathbf{W}^{[32]} \end{bmatrix}\right), \\ \mathbf{V}^{[2]} &\subseteq \text{null}\left(\begin{bmatrix} \mathbf{H}_2^{[11]} \mathbf{W}^{[11]} & \mathbf{H}_2^{[12]} \mathbf{W}^{[12]} & \mathbf{H}_2^{[31]} \mathbf{W}^{[31]} & \mathbf{H}_2^{[32]} \mathbf{W}^{[32]} \end{bmatrix}\right), \\ \mathbf{V}^{[3]} &\subseteq \text{null}\left(\begin{bmatrix} \mathbf{H}_3^{[11]} \mathbf{W}^{[11]} & \mathbf{H}_3^{[12]} \mathbf{W}^{[12]} & \mathbf{H}_3^{[21]} \mathbf{W}^{[21]} & \mathbf{H}_3^{[22]} \mathbf{W}^{[22]} \end{bmatrix}\right). \end{aligned}$$

Phase II: IUI elimination. We finally obtain an ICI-free channel, i.e., $\bar{\mathbf{H}}_j^{[jk]} = \mathbf{V}^{[j]\dagger} \mathbf{H}_j^{[jk]} \mathbf{W}^{[jk]}$, $j \in \{1, 2, 3\}$ and $k \in \{1, 2\}$. The j -th BS calculates $\mathbf{P}_1^{[j]} = \text{null}(\bar{\mathbf{H}}_j^{[j2]})$, $\mathbf{P}_2^{[j]} = \text{null}(\bar{\mathbf{H}}_j^{[j1]})$, and IUI is eliminated.

Then 18 streams can be sent simultaneously without any interference. If conventional schemes, such as orthogonal schemes are used, at most 16 DoF is achievable. For general antenna configuration and general number of users per cell in a three-cell multi-user MIMO environment when $M > N$, we have the following theorem.

Theorem 1: In a three-cell constant interfering network, we assume that each BS is equipped with M antennas, each MS is equipped with N antennas, K cell-edge users are served simultaneously per cell, and each user sends d streams to its served BS. If $M = KN$ and $d \leq \lfloor \frac{M}{3K-1} \rfloor$ or if $M < KN$ and $d \leq \min\{\lfloor \frac{M}{3K-1} \rfloor, 3(KN - M)\}$, then $\eta = 3Kd$ DoF is achievable.

Proof: When $M = KN$ and $d \leq \lfloor \frac{M}{3K-1} \rfloor$, we combine all cell-edge users into an effective user. Let $\mathbf{G}^{[ij]} = [\mathbf{H}_i^{[j1]} \ \dots \ \mathbf{H}_i^{[jK]}]$ be the combined channel matrix. Let $\mathbf{W}^{[i]} = [\mathbf{W}^{[i1]\dagger} \ \dots \ \mathbf{W}^{[iK]\dagger}]^\dagger$ represent the combined transmit precoding matrix. Then the effective channel is a three-user IC

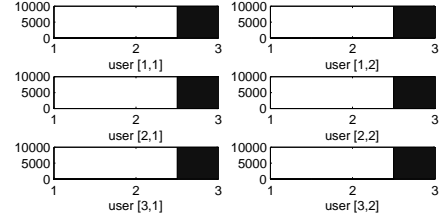


Fig. 3. Rank distribution of $\mathbf{W}^{[ij]}$ at user $[i, j]$ for the motivating example, where 10000 Monte Carlo tests are performed.

where each node is equipped with M antennas. Following the analysis in [1], there exists a $KN \times d$ $\mathbf{W}^{[i]}$, $i = \{1, 2, 3\}$, satisfies (22) - (24) as $d \leq \lfloor \frac{M}{3K-1} \rfloor < \frac{M}{2}$.

$$\text{span}(\mathbf{G}^{[12]} \mathbf{W}^{[2]}) = \text{span}(\mathbf{G}^{[13]} \mathbf{W}^{[3]}), \quad (22)$$

$$\text{span}(\mathbf{G}^{[21]} \mathbf{W}^{[1]}) = \text{span}(\mathbf{G}^{[23]} \mathbf{W}^{[3]}), \quad (23)$$

$$\text{span}(\mathbf{G}^{[31]} \mathbf{W}^{[1]}) = \text{span}(\mathbf{G}^{[32]} \mathbf{W}^{[2]}). \quad (24)$$

Let $\mathbf{W}^{[ij]} = [\mathbf{w}_1^{[ij]} \ \dots \ \mathbf{w}_d^{[ij]}]$, $i \in \{1, 2, 3\}$ and $j \in \{1, \dots, K\}$. We rewrite (22) as

$$\begin{aligned} & \text{span}\left(\begin{bmatrix} \mathbf{H}_1^{[21]} & \dots & \mathbf{H}_1^{[2K]} \end{bmatrix} \begin{bmatrix} \mathbf{w}_k^{[21]} \\ \vdots \\ \mathbf{w}_k^{[2K]} \end{bmatrix}\right) \\ &= \text{span}\left(\begin{bmatrix} \mathbf{H}_1^{[31]} & \dots & \mathbf{H}_1^{[3K]} \end{bmatrix} \begin{bmatrix} \mathbf{w}_k^{[31]} \\ \vdots \\ \mathbf{w}_k^{[3K]} \end{bmatrix}\right), \end{aligned} \quad (25)$$

where $k \in \{1, \dots, d\}$, i.e.,

$$\begin{aligned} & \text{span}(\mathbf{H}_1^{[21]} \mathbf{w}_k^{[21]} + \dots + \mathbf{H}_1^{[2K]} \mathbf{w}_k^{[2K]}) \\ &= \text{span}(\mathbf{H}_1^{[31]} \mathbf{w}_k^{[31]} + \dots + \mathbf{H}_1^{[3K]} \mathbf{w}_k^{[3K]}). \end{aligned} \quad (26)$$

So, there exist non-zero α_{1k} and α_{2k} satisfy

$$\begin{aligned} & \alpha_{1k} (\mathbf{H}_1^{[21]} \mathbf{w}_k^{[21]} + \dots + \mathbf{H}_1^{[2K]} \mathbf{w}_k^{[2K]}) \\ &= \alpha_{2k} (\mathbf{H}_1^{[31]} \mathbf{w}_k^{[31]} + \dots + \mathbf{H}_1^{[3K]} \mathbf{w}_k^{[3K]}). \end{aligned} \quad (27)$$

i.e.,

$$\dim\left\{\text{span}\left(\begin{bmatrix} \mathbf{H}_1^{[21]} \mathbf{w}_k^{[21]} & \dots & \mathbf{H}_1^{[2K]} \mathbf{w}_k^{[2K]} \\ \mathbf{H}_1^{[31]} \mathbf{w}_k^{[31]} & \dots & \mathbf{H}_1^{[3K]} \mathbf{w}_k^{[3K]} \end{bmatrix}\right)\right\} = 2K - 1. \quad (28)$$

Then we have

$$\begin{aligned} & \dim\left\{\text{span}\left(\begin{bmatrix} \mathbf{H}_1^{[21]} \mathbf{W}^{[21]} & \dots & \mathbf{H}_1^{[2K]} \mathbf{W}^{[2K]} \\ \mathbf{H}_1^{[31]} \mathbf{W}^{[31]} & \dots & \mathbf{H}_1^{[3K]} \mathbf{W}^{[3K]} \end{bmatrix}\right)\right\} \\ &= \dim\left\{\text{span}\left(\begin{bmatrix} \mathbf{H}_1^{[21]} \mathbf{w}_1^{[21]} & \dots & \mathbf{H}_1^{[2K]} \mathbf{w}_1^{[2K]} \\ \mathbf{H}_1^{[31]} \mathbf{w}_1^{[31]} & \dots & \mathbf{H}_1^{[3K]} \mathbf{w}_1^{[3K]} \end{bmatrix}\right)\right\} \\ &+ \dots \\ &+ \dim\left\{\text{span}\left(\begin{bmatrix} \mathbf{H}_1^{[21]} \mathbf{w}_d^{[21]} & \dots & \mathbf{H}_1^{[2K]} \mathbf{w}_d^{[2K]} \\ \mathbf{H}_1^{[31]} \mathbf{w}_d^{[31]} & \dots & \mathbf{H}_1^{[3K]} \mathbf{w}_d^{[3K]} \end{bmatrix}\right)\right\} \end{aligned}$$

$$\bar{\mathbf{H}} = \begin{bmatrix} 0 & \dots & 0 & \mathbf{H}_1^{[21]} & \dots & \mathbf{H}_1^{[2K]} & \mathbf{H}_1^{[31]} & \dots & \mathbf{H}_1^{[3K]} \\ \mathbf{H}_2^{[11]} & \dots & \mathbf{H}_2^{[1K]} & 0 & \dots & 0 & \mathbf{H}_2^{[31]} & \dots & \mathbf{H}_2^{[3K]} \\ \mathbf{H}_3^{[11]} & \dots & \mathbf{H}_3^{[1K]} & \mathbf{H}_3^{[21]} & \dots & \mathbf{H}_3^{[2K]} & 0 & \dots & 0 \end{bmatrix} \quad (32)$$

$$\left. \mathbf{H}_1^{[31]} \mathbf{w}_d^{[31]} \dots \mathbf{H}_1^{[3K]} \mathbf{w}_d^{[3K]} \right\} \\ = (2K-1)d. \quad (29)$$

So, the interference space at BS1 can be aligned into a vector space of $(2K-1)d$ dimensions. Along the same way, we also have

$$\dim \left\{ \text{span} \left(\left[\mathbf{H}_2^{[11]} \mathbf{W}^{[11]} \dots \mathbf{H}_2^{[1K]} \mathbf{W}^{[1K]} \mathbf{H}_2^{[31]} \mathbf{W}^{[31]} \dots \mathbf{H}_2^{[3K]} \mathbf{W}^{[3K]} \right] \right) \right\} = (2K-1)d, \quad (30)$$

$$\dim \left\{ \text{span} \left(\left[\mathbf{H}_3^{[11]} \mathbf{W}^{[11]} \dots \mathbf{H}_3^{[1K]} \mathbf{W}^{[1K]} \mathbf{H}_3^{[21]} \mathbf{W}^{[21]} \dots \mathbf{H}_3^{[2K]} \mathbf{W}^{[2K]} \right] \right) \right\} = (2K-1)d. \quad (31)$$

Then, each BS needs a vector space of $M \geq [(2K-1) + K]d$ dimensions to decode Kd streams sent from its K cell-edge users, i.e., $d \leq \lfloor \frac{M}{3K-1} \rfloor$ should be satisfied when M and K are given. Then let $\mathbf{V}^{[i]}$, $i \in \{1, 2, 3\}$, be a subset of (or equal to) the null space of the interference at each BS, which is a $M \times Kd$ matrix. IUI elimination matrix can be calculated accordingly. So, if $M = KN$ and $d \leq \lfloor \frac{M}{3K-1} \rfloor$, totally $3Kd$ DoF is achievable.

When $M < KN$ and $d \leq \min \left\{ \lfloor \frac{M}{3K-1} \rfloor, 3(KN - M) \right\}$, We combine all the interference signals into a matrix $\bar{\mathbf{H}}$ which is defined as (32). Let

$$\bar{\mathbf{W}} = \left[\mathbf{W}^{[11]\dagger} \dots \mathbf{W}^{[1K]\dagger} \mathbf{W}^{[21]\dagger} \dots \mathbf{W}^{[2K]\dagger} \mathbf{W}^{[31]\dagger} \dots \mathbf{W}^{[3K]\dagger} \right]^\dagger, \quad (33)$$

and let

$$\bar{\mathbf{W}} \subseteq \text{null}(\bar{\mathbf{H}}). \quad (34)$$

The dimension of $\bar{\mathbf{H}}$ is $3M \times 3KN$, and the dimension of the null space of $\bar{\mathbf{H}}$ is $3(KN - M)$. Let $d \leq 3(KN - M)$, then there exists a $3KN \times d$ $\bar{\mathbf{W}}$ satisfies (34). Then (29) - (31) are also satisfied. By using the same argument as in the motivating example, $\mathbf{W}^{[ij]}$ will be full rank with probability 1.

The dimension of the interference space at each BS is decreased to $(2K-1)d$, then each BS needs a vector space of $M \geq [(2K-1) + K]d$ dimensions to decode Kd streams sent from its K cell-edge users, i.e., $d \leq \lfloor \frac{M}{3K-1} \rfloor$ should be satisfied when M and K are given. ICI elimination matrix $\mathbf{V}^{[i]}$ and IUI elimination matrix $\mathbf{P}^{[i]}$, $i \in \{1, 2, 3\}$, can be calculated accordingly. Then totally $3Kd$ streams can be sent simultaneously, i.e., when $M < KN$ and $d \leq \min \left\{ \lfloor \frac{M}{3K-1} \rfloor, 3(KN - M) \right\}$, $3Kd$ DoF is achievable. For example, when $M = 8$, $N = 4$, and $K = 3$, one stream can be sent from each user simultaneously. Then $\eta = 9$ is achievable, while if orthogonal schemes are used, at most 8 interference-free streams can be sent simultaneously. ■

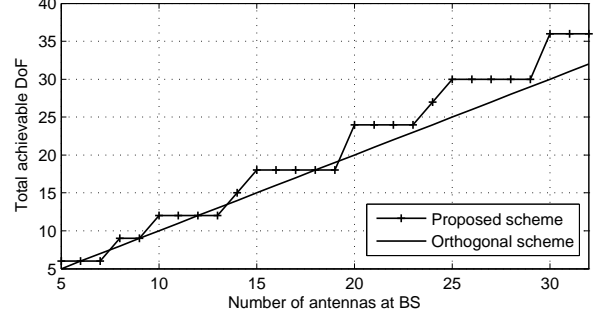


Fig. 4. The comparison of DoF achievable between our IA based scheme and orthogonal schemes when $N < M \leq KN$.

IV. NUMERICAL RESULTS

The comparison of DoF achievable between our scheme and orthogonal schemes is presented in Fig. 4 when $N < M \leq KN$ and

$$K = \arg \max_{K \in \mathbb{R}, N < M} 3K \cdot \min \left\{ \lfloor \frac{M}{3K-1} \rfloor, 3(KN - M) \right\},$$

where K varies from 2 to 5 when $M \leq 32$. It is shown that our scheme can achieve more DoF compared with orthogonal schemes in most cases. However, achievable DoF is less than orthogonal schemes when $M = 7, 13$, or 19 , as some dimensions are wasted at BSs. If symbol extensions are allowed, even with constant channel, it is expected that more DoF can be achieved than that of orthogonal schemes by using similar scheme in [8], and we leave it for future work.

REFERENCES

- [1] V. R. Cadambe and S. A. Jafar, "Interference alignment and the degrees of freedom for the K -user interference channel," *IEEE Trans. Inf. Theory*, vol. 54, no. 8, pp. 3425-3441, Aug. 2008.
- [2] R. Tresh, M. Guiland, and E. Riegler, "On the achievability of interference alignment in the K -user constant MIMO interference channel," in *Proc. IEEE Workshop Stat. Signal Process.*, pp. 277-280, Cardiff, U.K., Sep. 2009.
- [3] M. Razaviyayn, G. Lyubeznik, and Z.-Q. Luo, "On the degrees of freedom achievable through interference alignment in a MIMO interference channel," *IEEE Trans. Signal Process.*, vol. 60, no. 2, pp. 812-821, Feb. 2012.
- [4] G. Bresler, D. Cartwright, and D. Tse, "Settling the feasibility of interference alignment for the MIMO interference channel: the symmetric case," in arXiv:1104.0888v1, Apr. 2011.
- [5] K. Gomadam, V. R. Cadambe, and S. A. Jafar, "Approaching the capacity of wireless networks through distributed interference alignment," in *Proc. of IEEE GLOBECOM*, Dec. 2008.
- [6] C. Suh, M. Ho, and D. Tse, "Downlink interference alignment," *IEEE Trans. Commun.*, vol. 59, no. 9, pp. 2616-2626, Sep. 2011.
- [7] W. Shin, N. Lee, J.-B. Lim, C. Shin, and K. Jang, "On the design of interference alignment scheme for two-cell MIMO interfering broadcast channels," *IEEE Trans. on Wireless Communications*, Vol. 10, no. 2, pp. 437-442, Feb. 2011.
- [8] P. Mohapatra, K. E. Nissar, and C. R. Murthy, "Interference alignment algorithms for the K user constant MIMO interference channel," *IEEE Trans. Signal Process.*, vol. 59, no. 11, pp. 5499-5508, Nov. 2011.